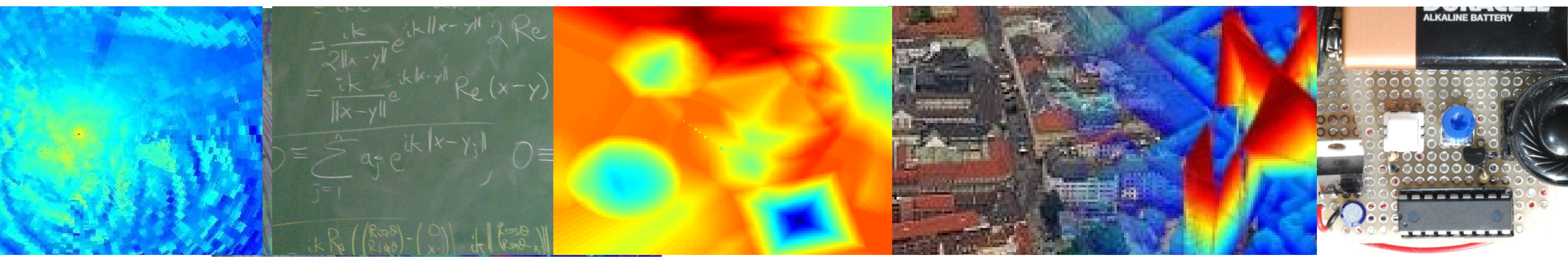
The Appearance *of* Stratified Spaces *in* Synthetic Aperture Sonar Collections



Michael Robinson

Acknowledgments

- Students:
  - Zander Memon (AU)
  - Harry Pham (AU)
  - Maxwell Gualtieri (Northwestern)
- Collaborator:
  - Brian DiZio (NUWC Newport)
- Data: ARL/PSU AirSAS and NUWC HFTAM
- Funding: Kyle Becker (ONR)

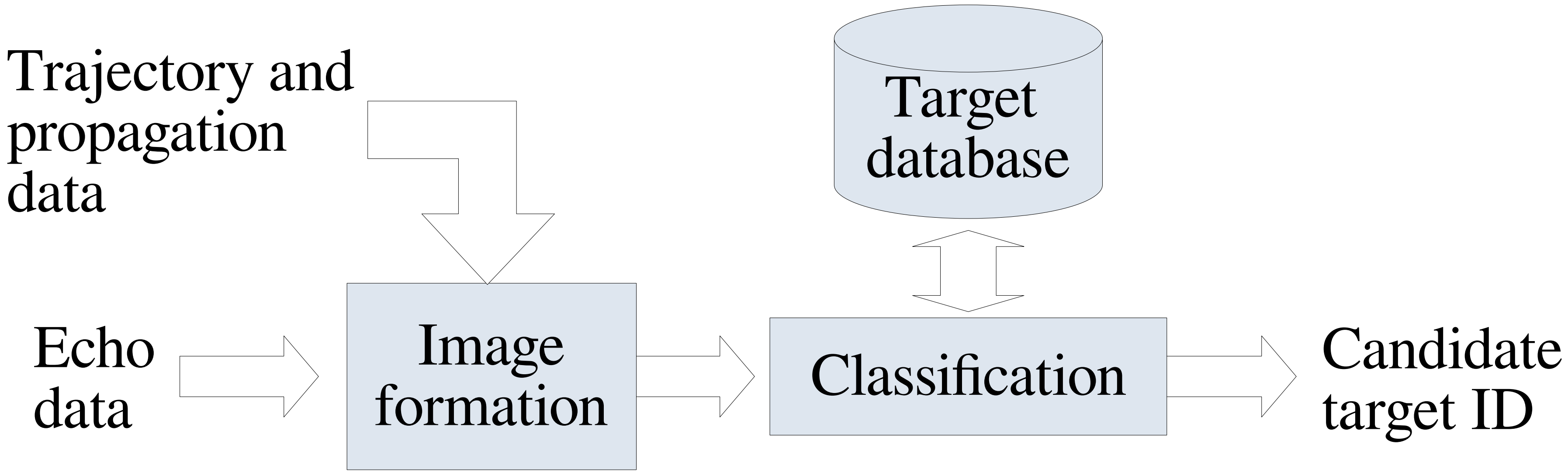
- Main references:

M. Robinson, Z. Memon, M. Gualtieri, “Topological and geometric characterization of synthetic aperture sonar collections.” *Journal Acoustic Society of America,* 2025. https://doi.org/10.1121/10.0037085

Z. Memon, M. Robinson, "The Topology of Circular Synthetic Aperture Sonar Targets." arXiv:2205.11311

Motivation

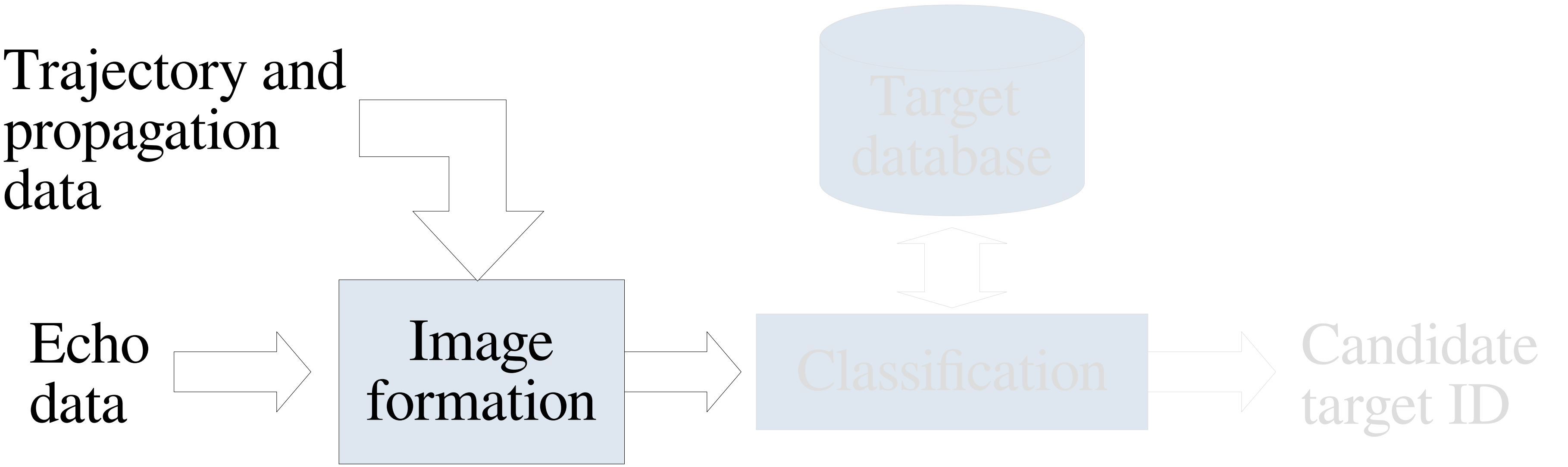
- Goal: Detect and identify objects on the seafloor using their active sonar signatures

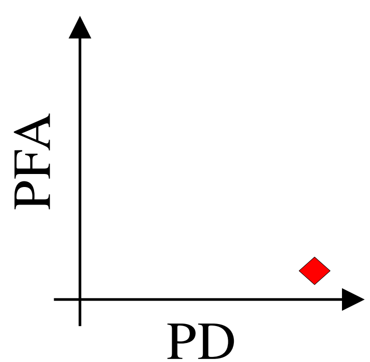


- Nearly all research, development, and fielded systems follow this common pipeline

Motivation

- Under tightly controlled conditions, sonar image quality is very good

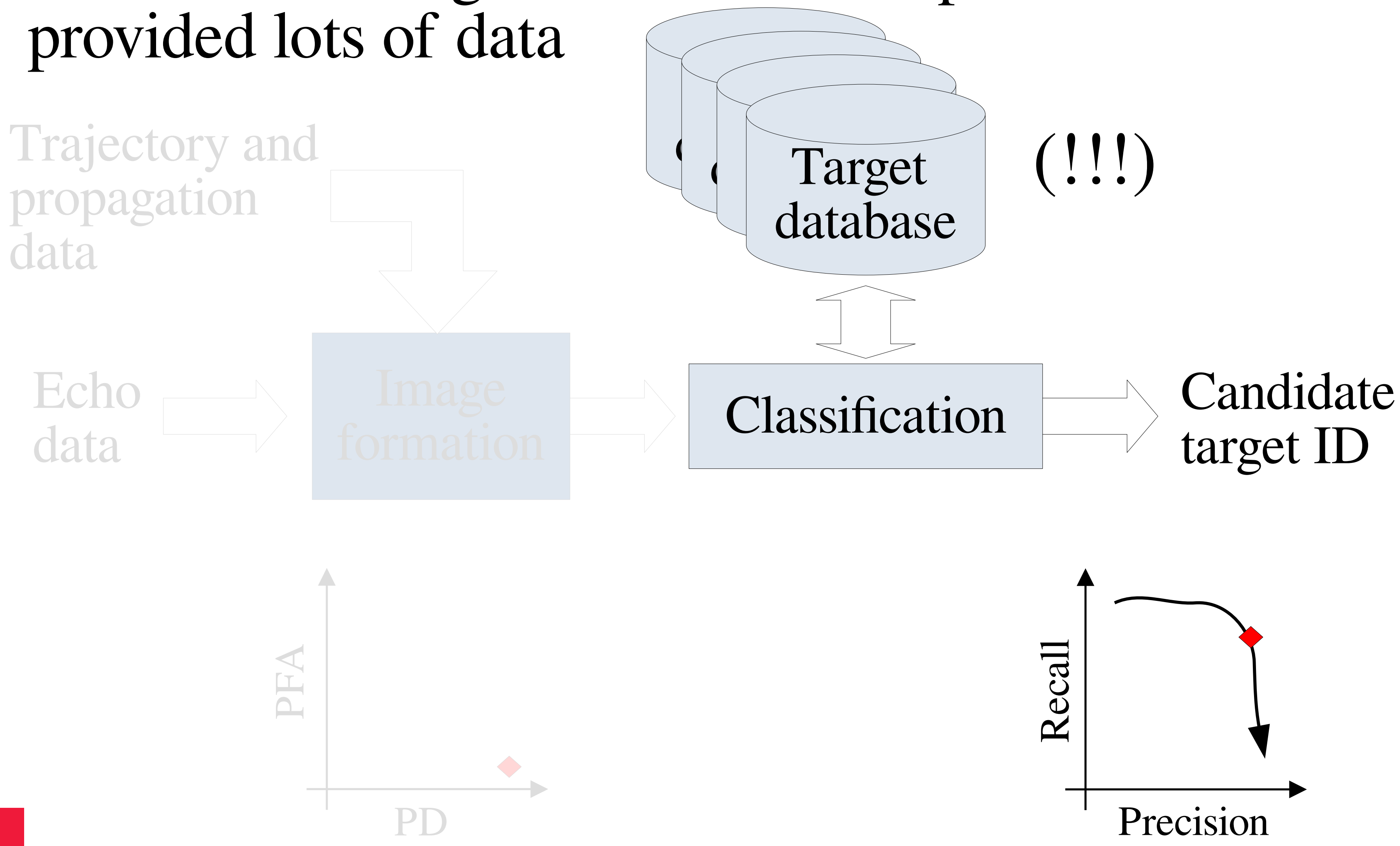




Target detection performance is good
Focus, contrast, etc. good too

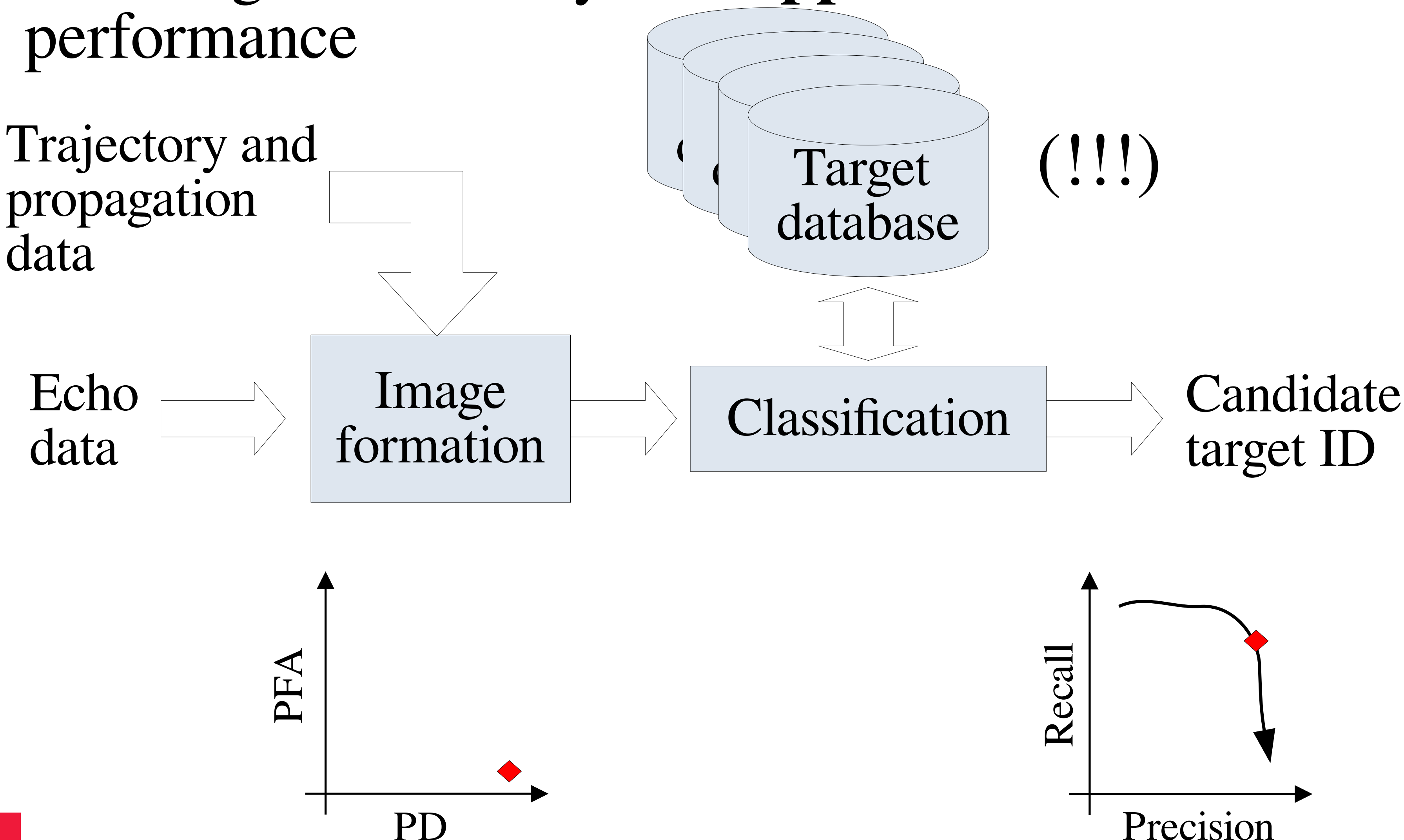
Motivation

- Machine learning-based classifiers perform well if provided lots of data



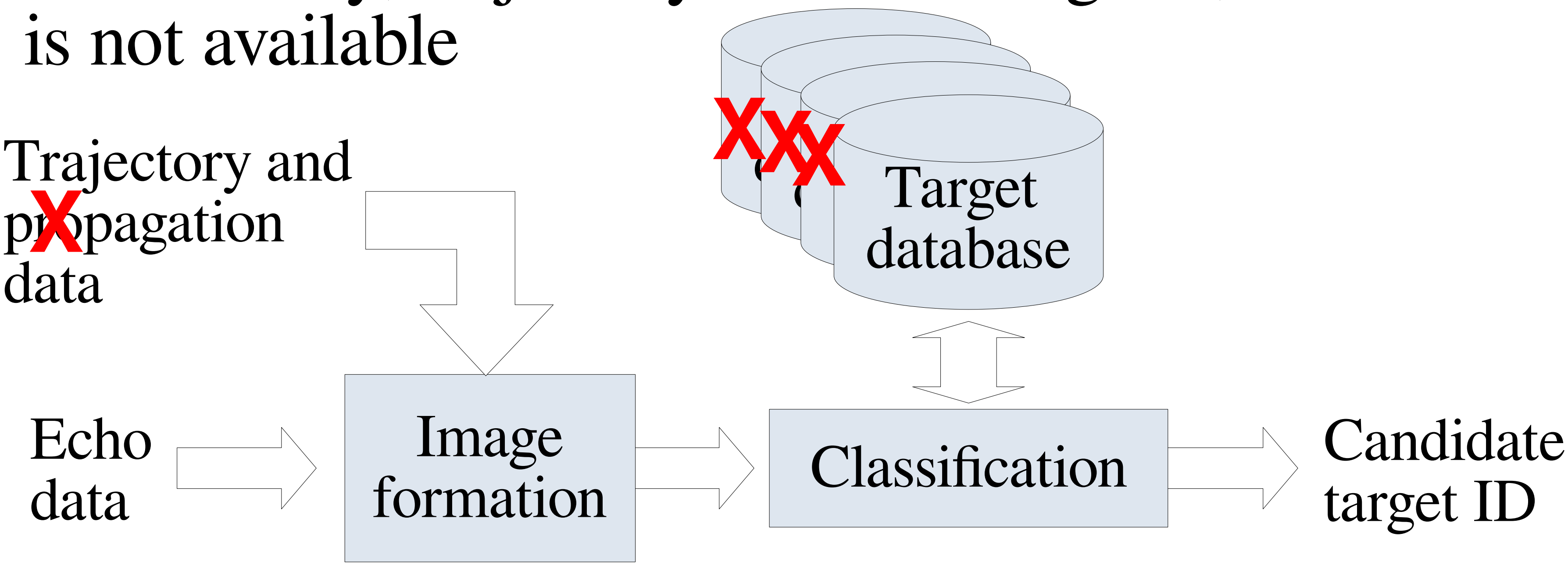
Motivation

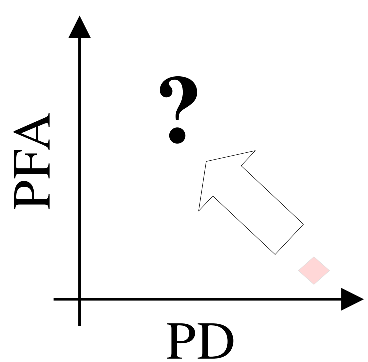
- Chaining these tools yields **upper bound** on overall performance

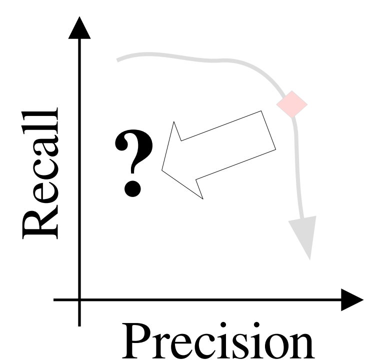


Motivation

- Realistically, trajectory won't be as good; vast training is not available

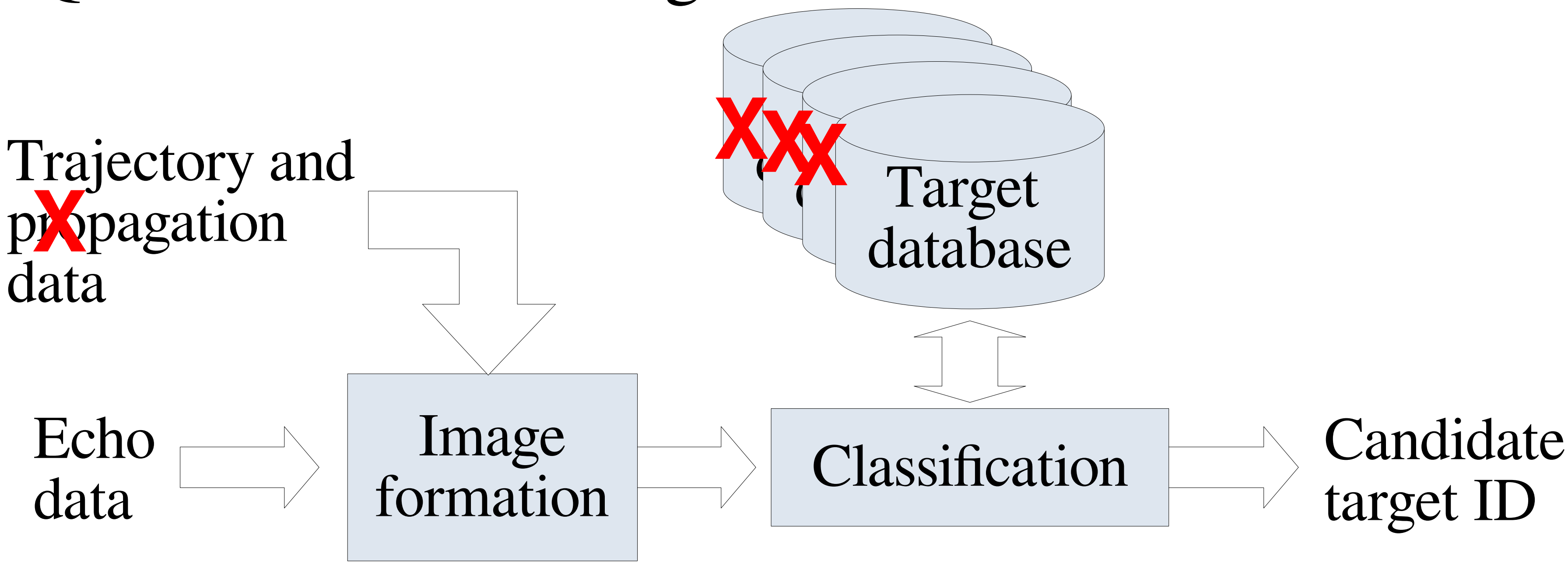


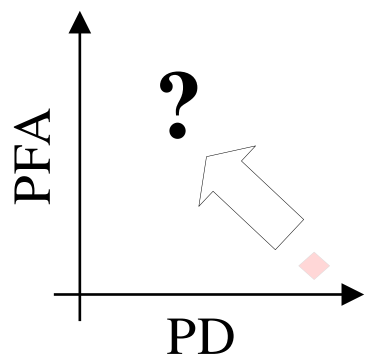


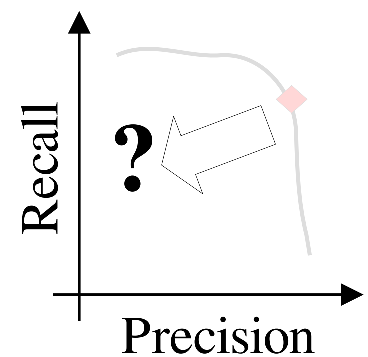


Motivation

- Question: What strategies remain in this case?

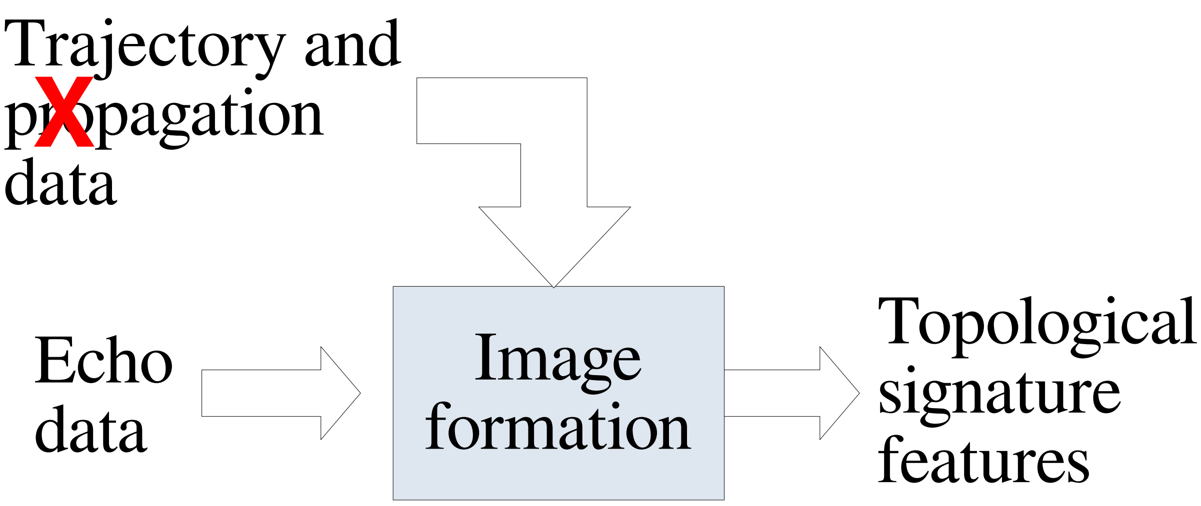


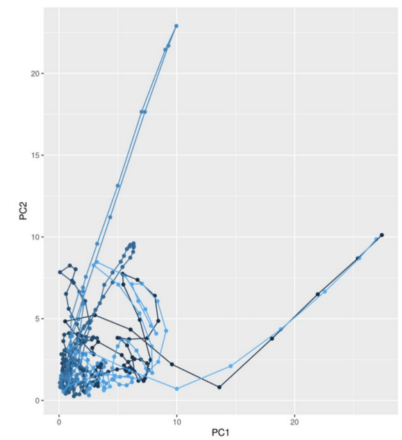


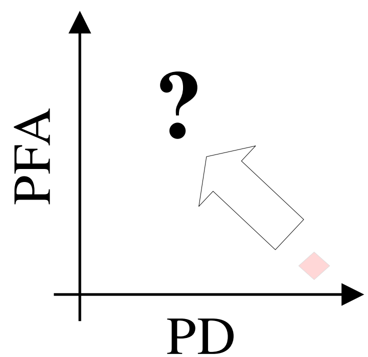


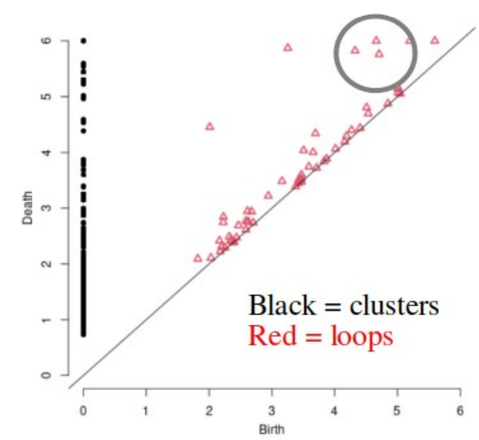
Motivation

- Answer: Trajectory-invariant topological signal rep'ns



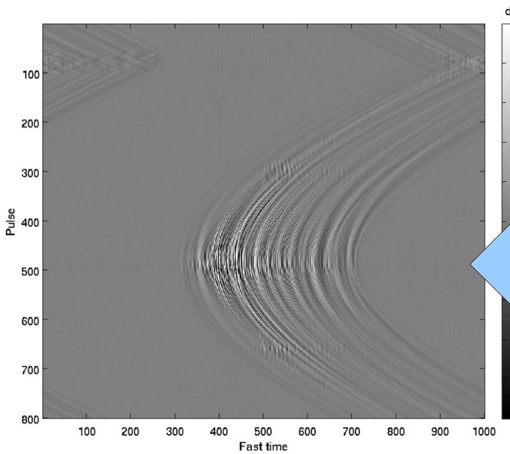




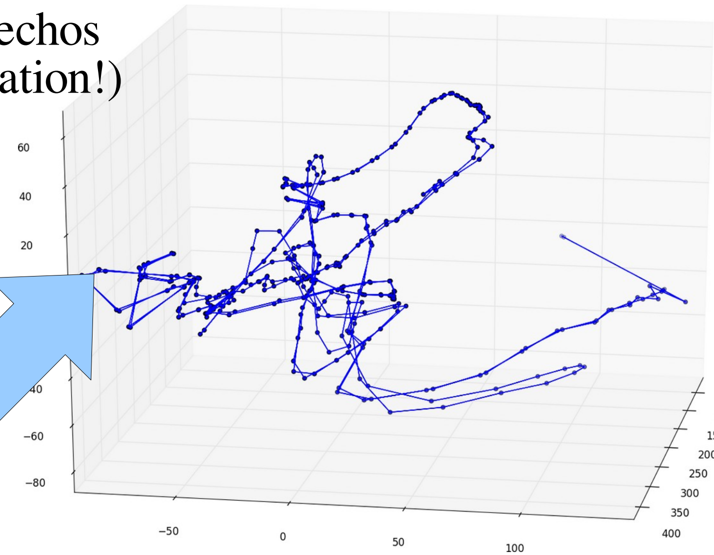


Initial insight: topological tools **do** work

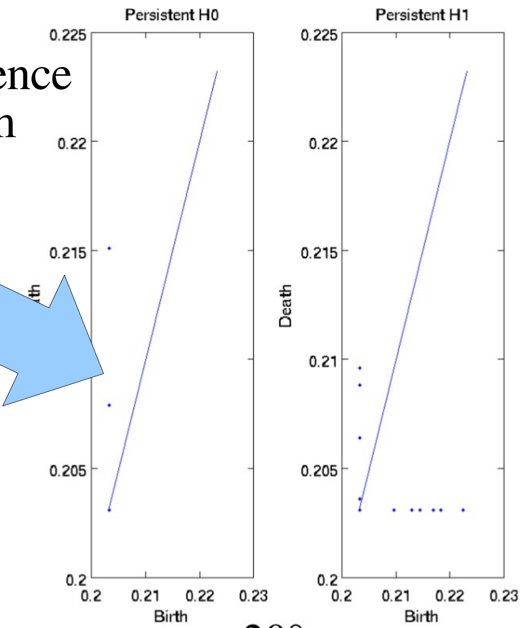
Pulses



Metric space of echos
(**not** image formation!)



Persistence diagram



(Processed data is representative only!)

20°

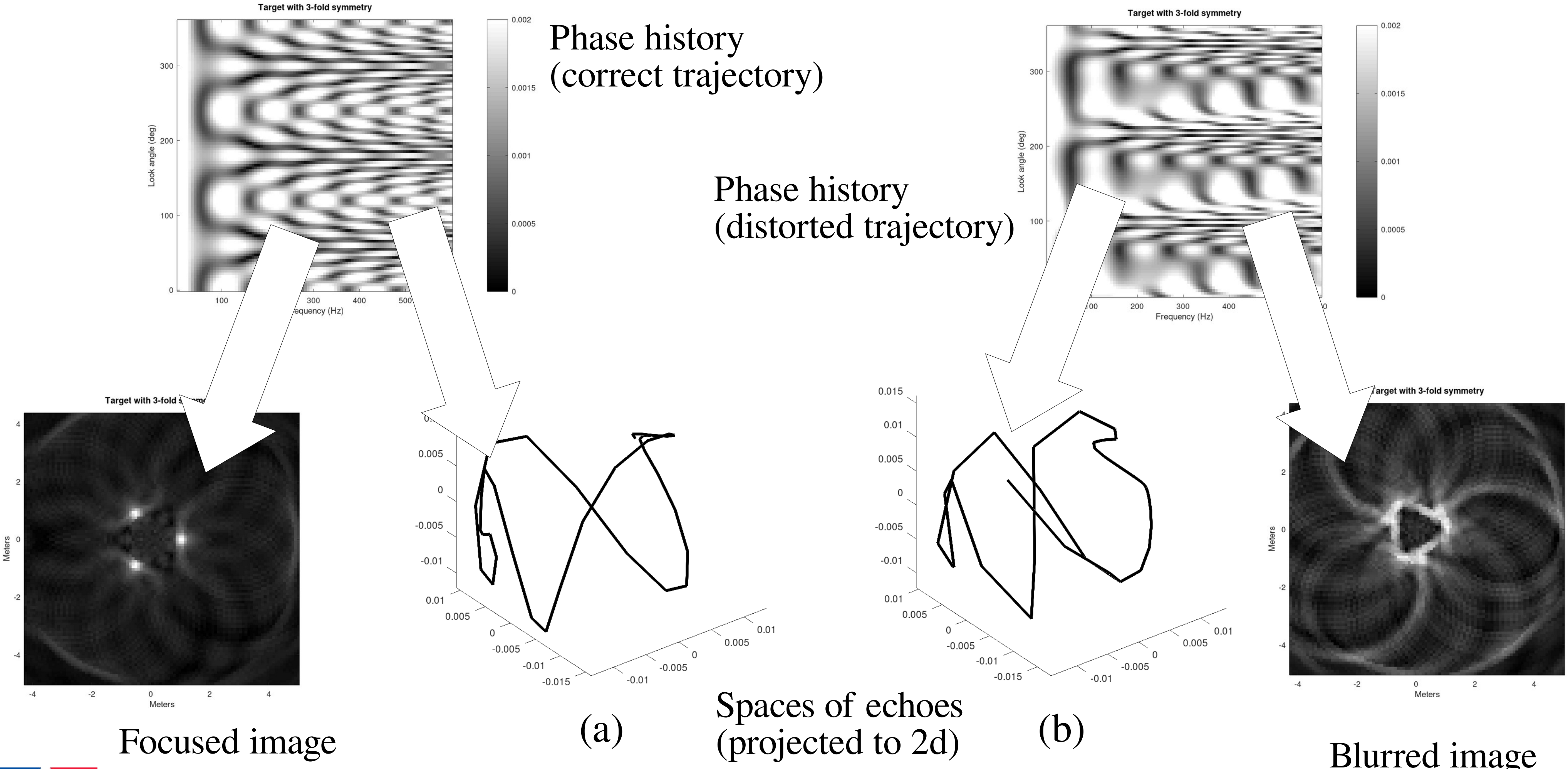
Table of mean misclassification rates for various metrics

(smaller is better)

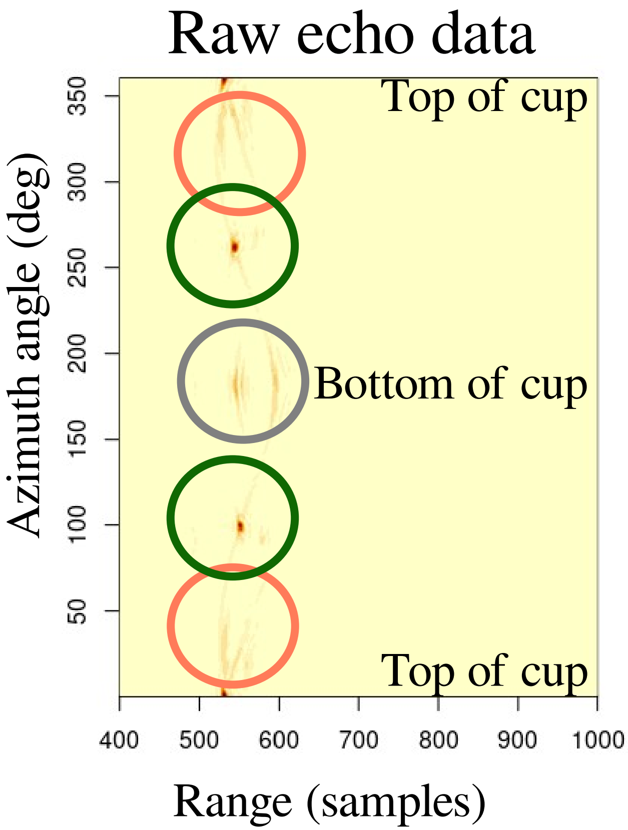
Metric	Target types	Target groups	
Spectral L^2	4.32	2.32	
Spectral corr.	3.58	2.19	
Tucker [6, Fig. 5(b)]	2.15	1.57	Non-topological
H_0 with L^2	**1.47**	**1.28**	Topological
H_1 with L^2	2.49	1.70	
H_0 with corr.	1.62	1.30	
H_1 with corr.	2.38	1.57	

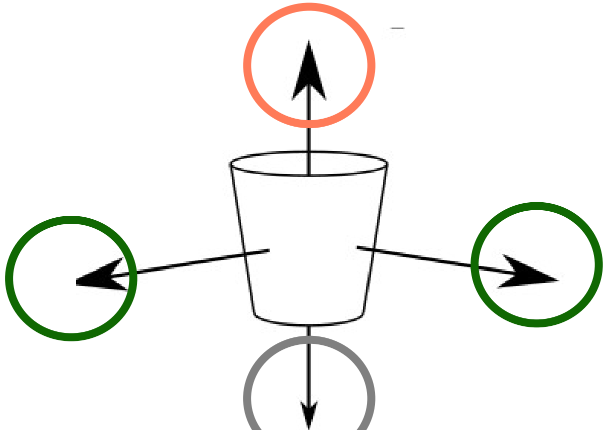
Topological features are robust

- The space of echos is ignores trajectory distortions

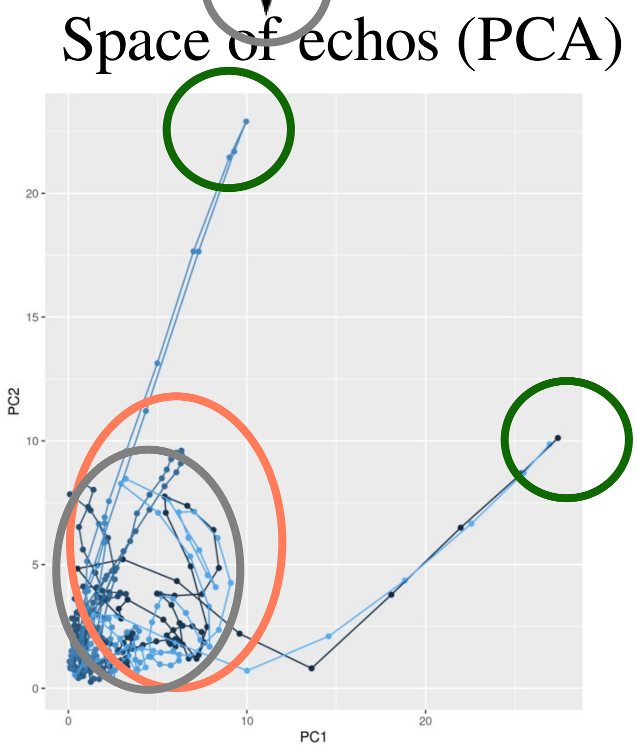


New insight: Prominent scatterers





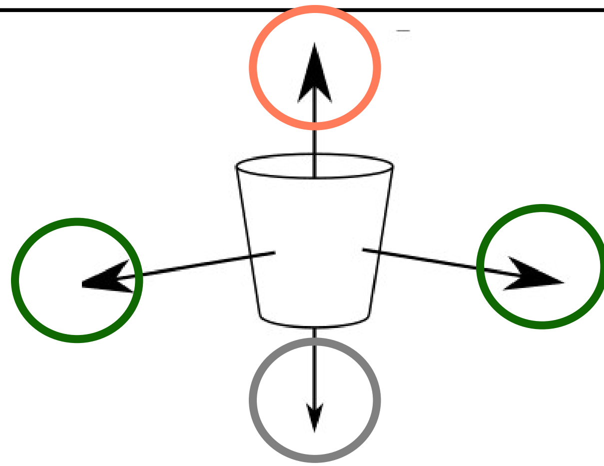
The top and bottom have weaker, wider beamwidth echos → wider, smaller loops



Prominent scatterer: Specific acoustic modality is isolated in the configuration space (phase history domain)

Isolated scatterers correspond to visible features in the space of echos

Topological space of echos



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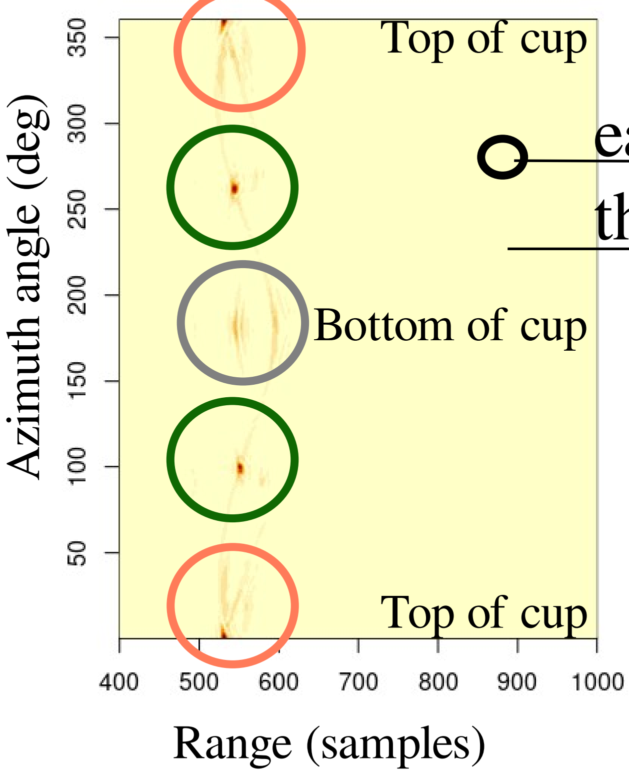
Prominent scatterer: Specific acoustic modality is isolated in the configuration space (phase history domain)

Raw echo data



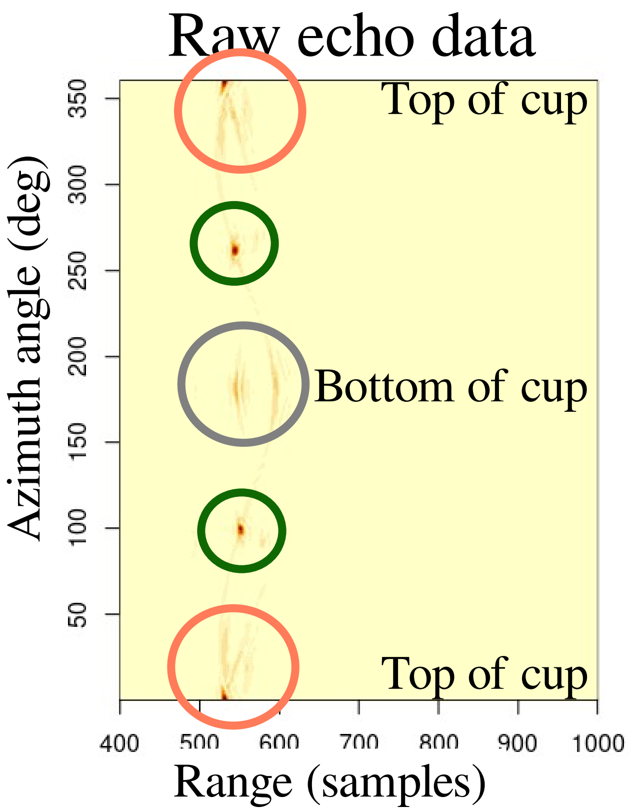
each measurement

the entire domain

$C^{\infty}_{D}(M, \mathbb{R}^n)$: the set of smooth functions defined on M, taking values in $\mathbb{R}^n$, supported on D.

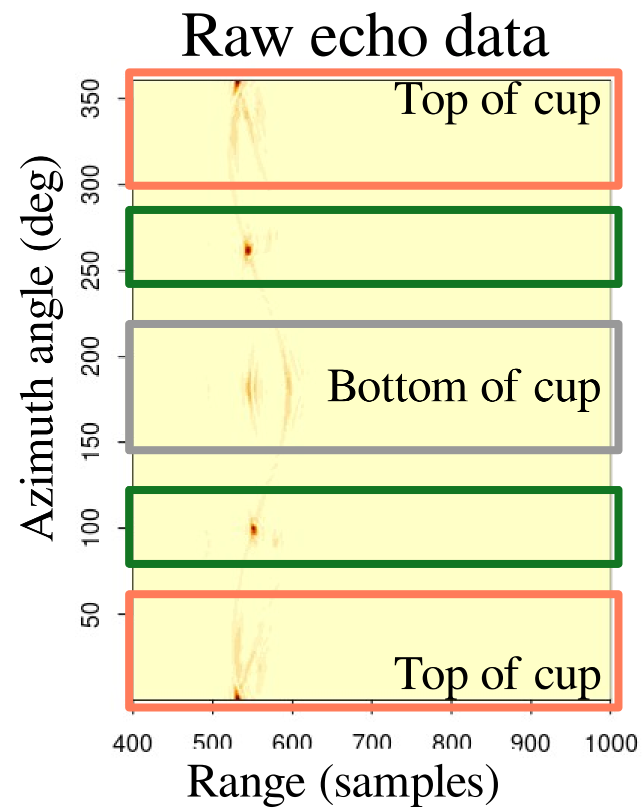
the set of prominent scatterers

Flexibility in the signal models



Topology says:

Both models equally valid… but one might be more useful!



$$C^{\infty}_{D}(S^1 \times \mathbb{R}, \mathbb{R})$$

azimuth, range, single sample

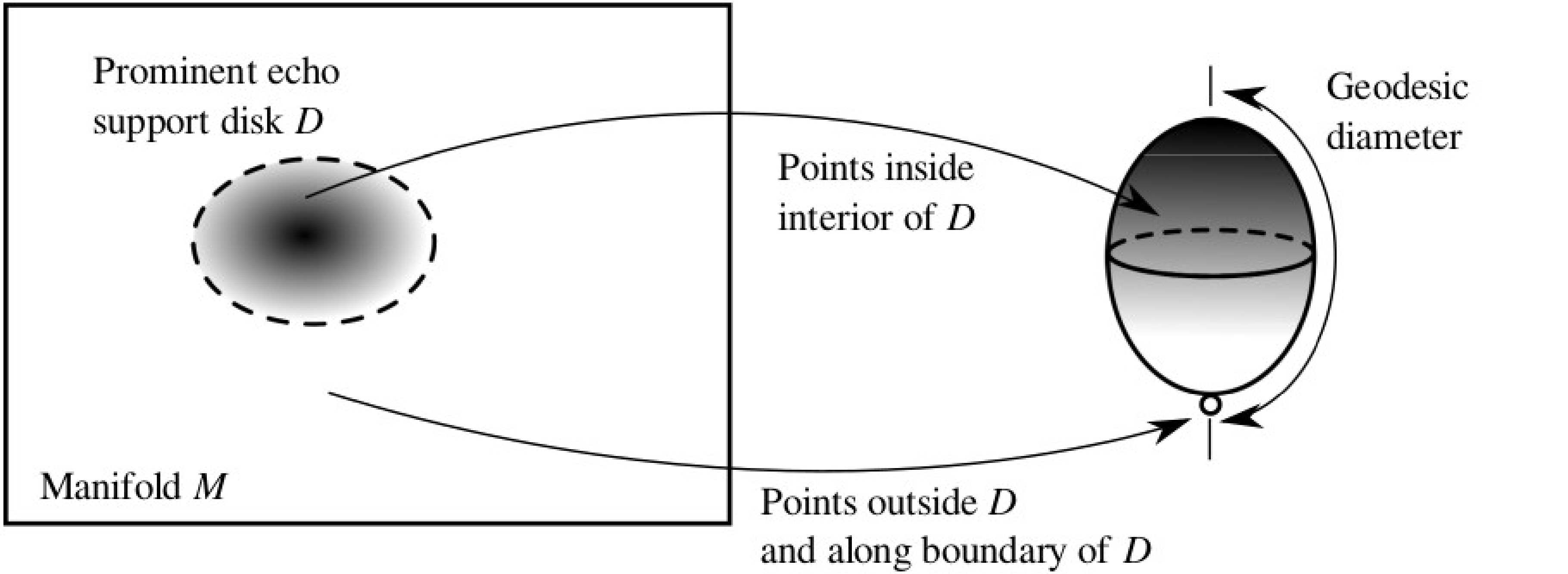
$$C^{\infty}_{D'}(S^1, \mathbb{R}^n)$$

azimuth, vector of range samples

Signals to spaces (our main result)

Theorem 1. *Let M be a smooth manifold and suppose that $n > 2\dim M$. For a generic v in $C^\infty_{\mathcal{D}}(M,\mathbb{R}^n)$, the signature space of v is homeomorphic to a wedge product of spheres of (intrinsic) dimension $\dim M$.*

Moreover, each prominent echo corresponds to a sphere of the same dimension as M in the signature space. Under the usual metric for $\mathbb{R}^n$, the cross section for a prominent echo is a lower bound for the geodesic diameter of its corresponding sphere[3].



A sonar signature is a function on M

Theorem 1. *Let M be a smooth manifold and suppose that $n > 2 \dim M$. For a generic v in $C^\infty_{\mathcal{D}}(M, \mathbb{R}^n)$, the signature space of v is homeomorphic to a wedge product of sp es of (intrinsic) dimension $\dim M$.*

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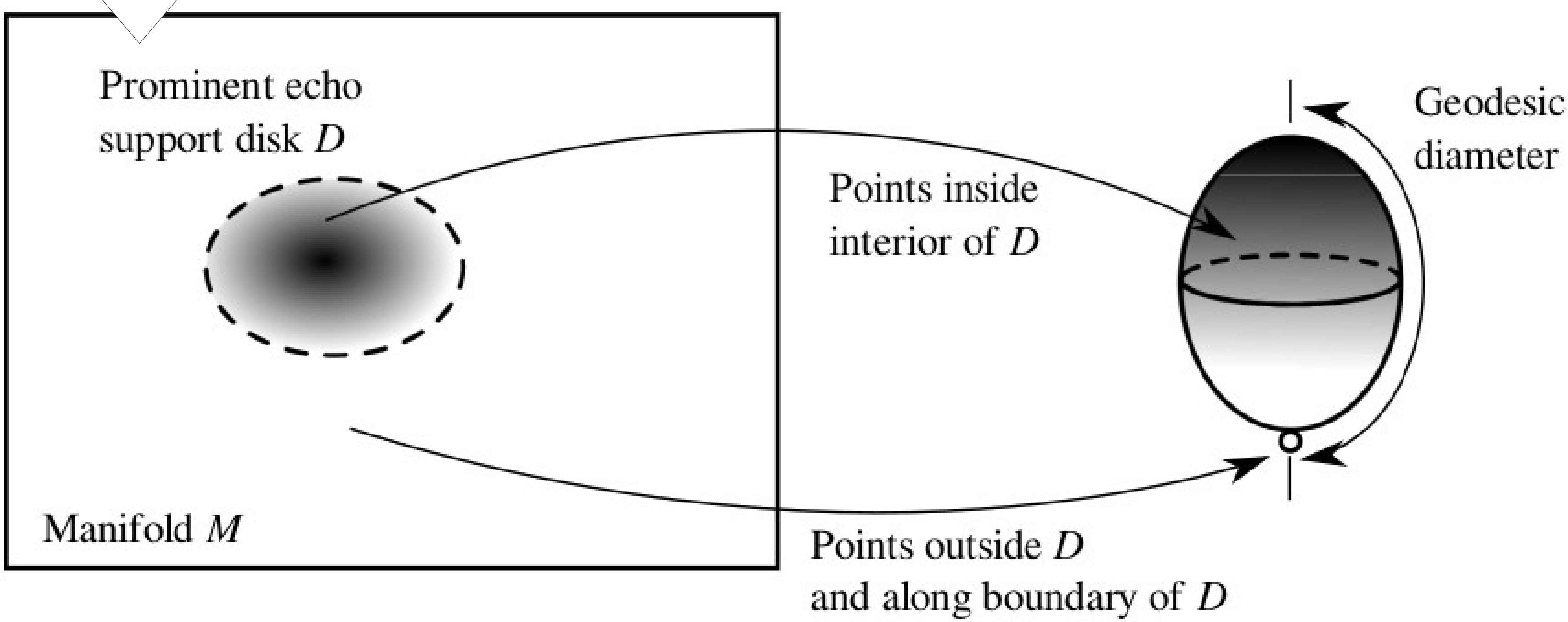
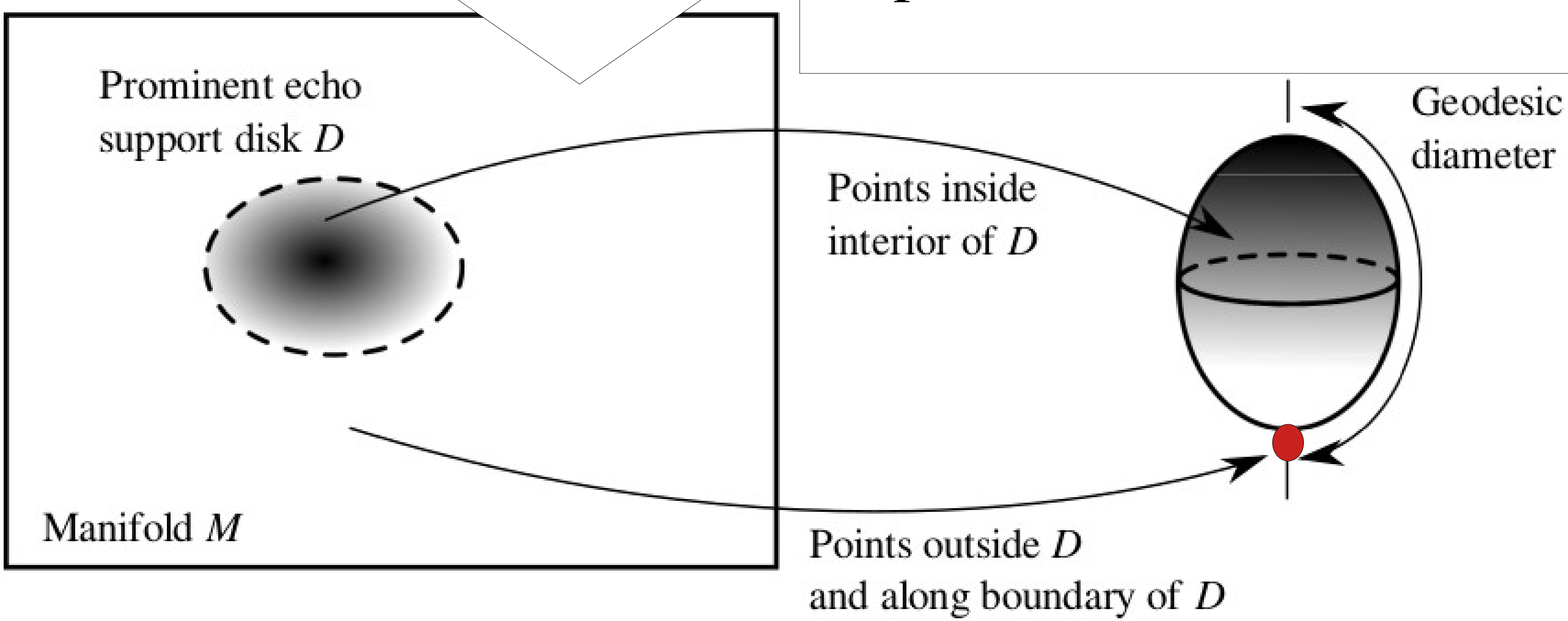


Its image is a "bouquet" of spheres

Theorem 1. *Let M be a smooth manifold and suppose that $n > 2\dim M$. For a generic v in $C^\infty_D(M,\mathbb{R}^n)$, the signature space of v is homeomorphic to a wedge product of spheres of (intrinsic) dimension $\dim M$.*

Moreover, each prominent … corresponds to a sphere of the same dimension as M in the signature sp… Under … oss section for a prominent echo i… ower bo… its corresponding sphere[3].

Every point outside prominent scatterers yields no echo, so maps to the same value



…with typically enough room to not overlap

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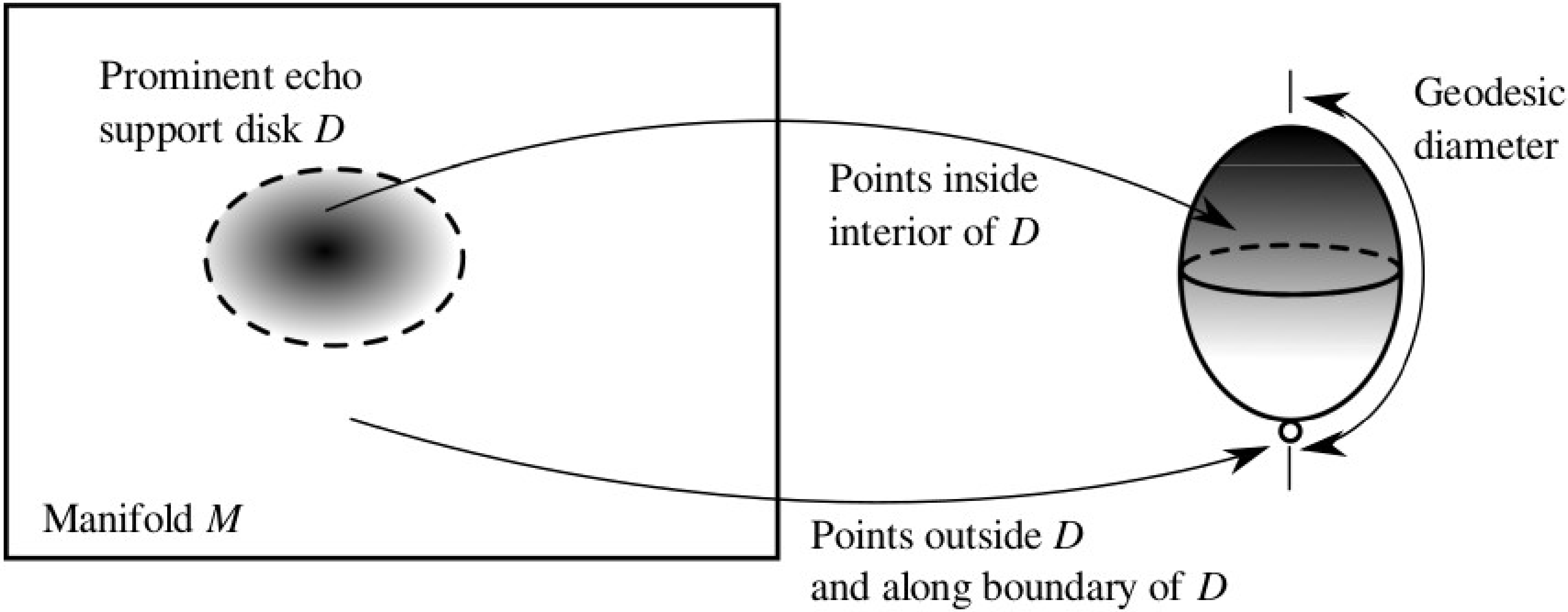
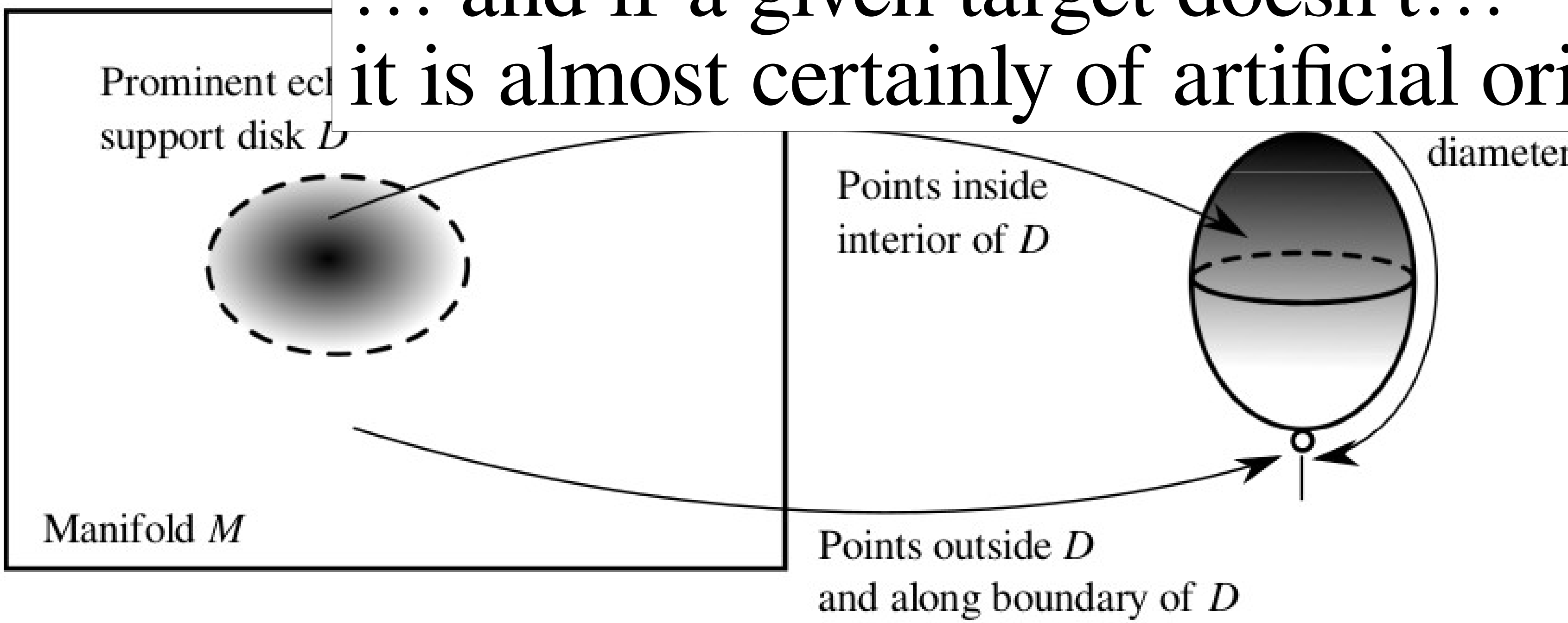


Typical targets will look like this

Theorem 1. *Let M be a smooth manifold and suppose that $n > 2 \dim M$. For a generic v in $C^\infty_D(M, \mathbb{R}^n)$, the signature space of v is homeomorphic to a wedge product of spheres of (intrinsic) dimension $\dim M$.*

Moreover, each prominent echo corresponds to a sphere of the same dimension as M in the signature space. Under the usual metric for $\mathbb{R}^n$, the cross section for a prominent echo is a lower bound for the geodesic diameter of its corresponding sphere[3].

… and if a given target doesn't…
it is almost certainly of artificial origin



Testing this as a null hypothesis

- Homeomorphisms cannot be tested directly
- But a weaker test is homology, and that we can compute directly from the theorem:

measurable!

$$\dim H_k(v(M)) = \begin{cases} 1 & \textit{if } k = 0, \\ \#\mathcal{D} & \textit{if } k = \dim M, \\ 0 & \textit{otherwise.} \end{cases}$$

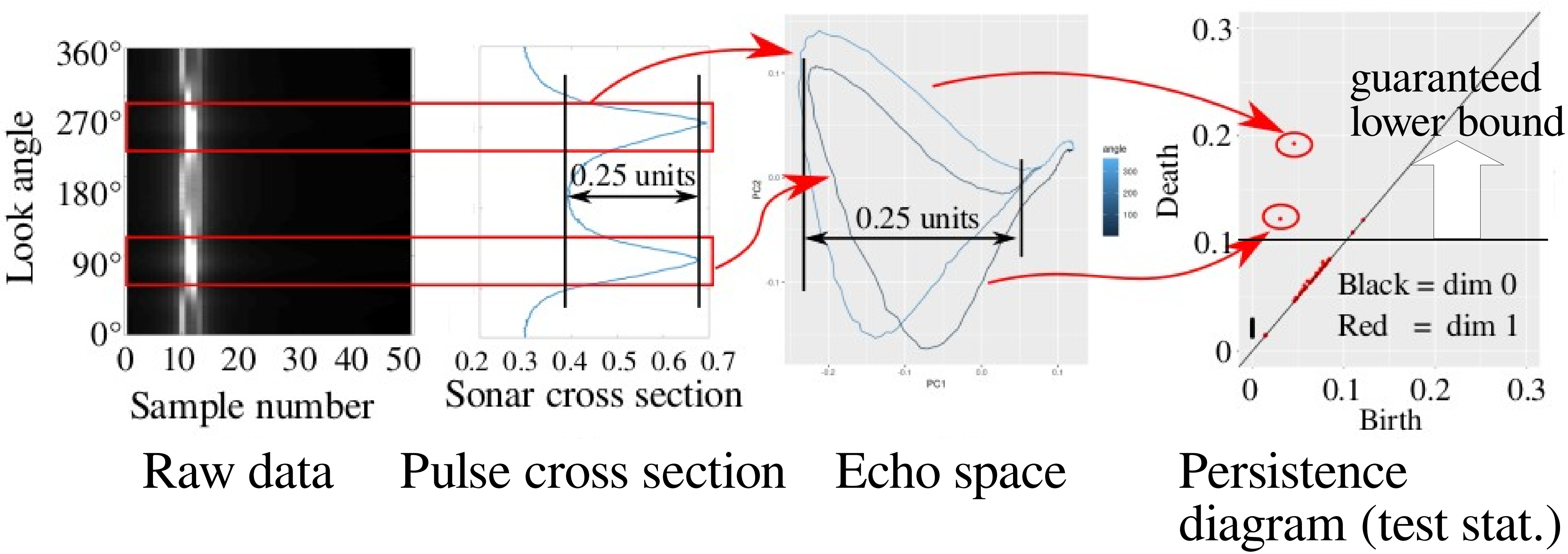
known!

Estimable via *persistent homology*

- Under noisy conditions, persistent homology is a test statistic* for "target unusualness"
- <u>Takeaway</u>: We **finally** have theoretical justification for using topology to classify!

*Omer Bobrowski and Primoz Skraba. A universal null-distribution for topological data analysis. *Scientific Reports*, 13(1):12274, 2023.

Simulation check: point scatterers



Proposition 7. *For the 1-dimensional embedded Vietoris-Rips filtration* VR_ϵ *constructed from the signature space as above, each prominent echo corresponds to a generator with death time bounded below by* $\sigma/2$.

0.25 / 2 = 0.125 in this case

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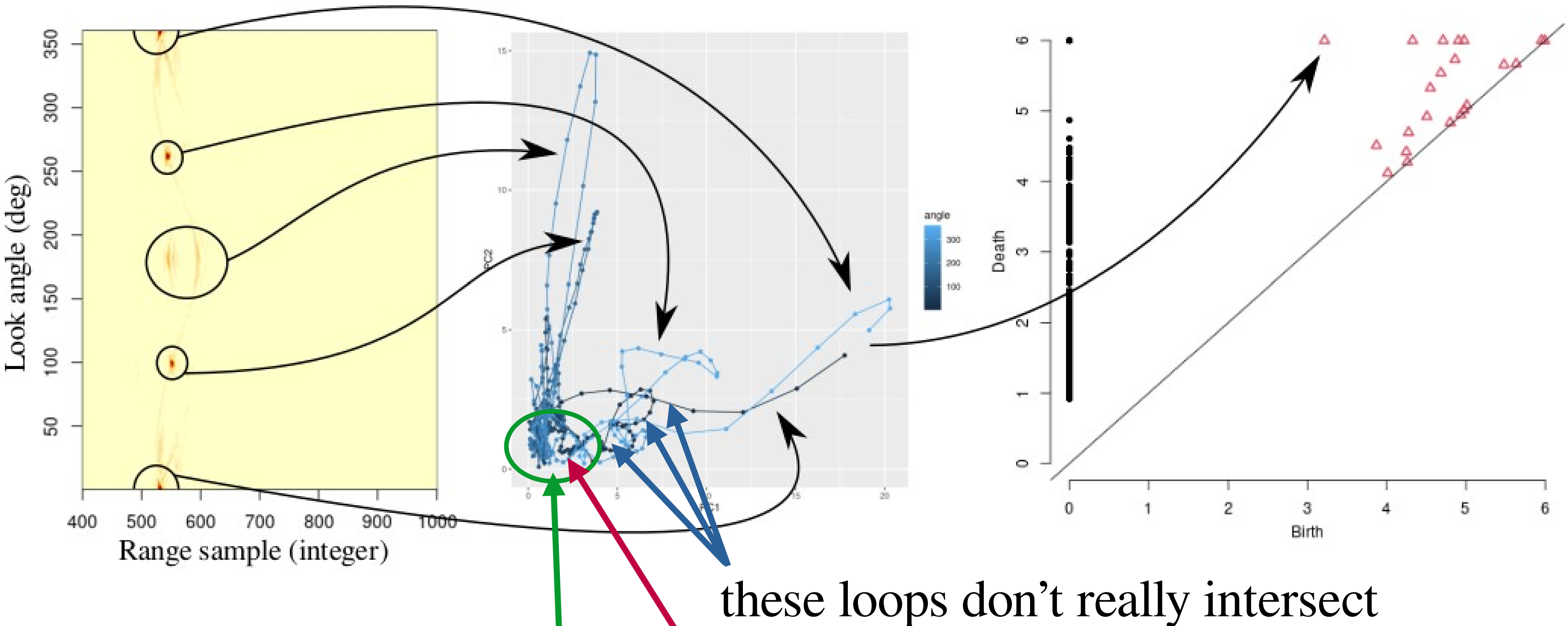
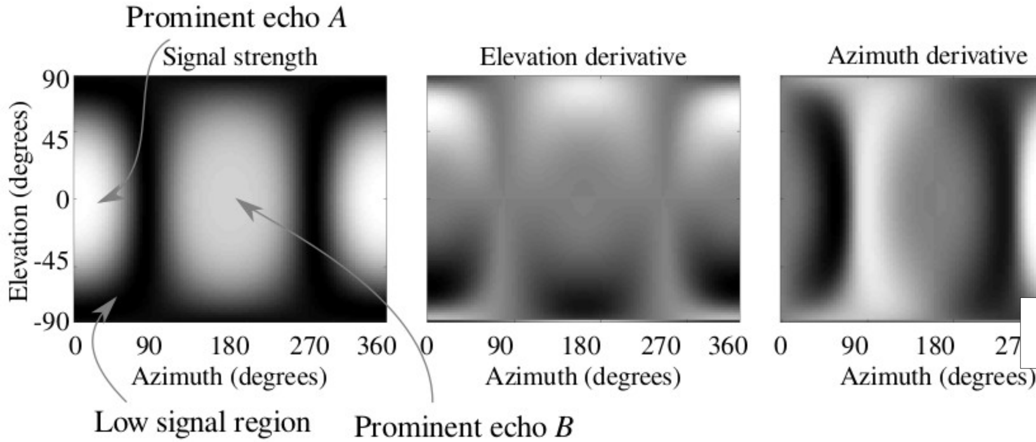
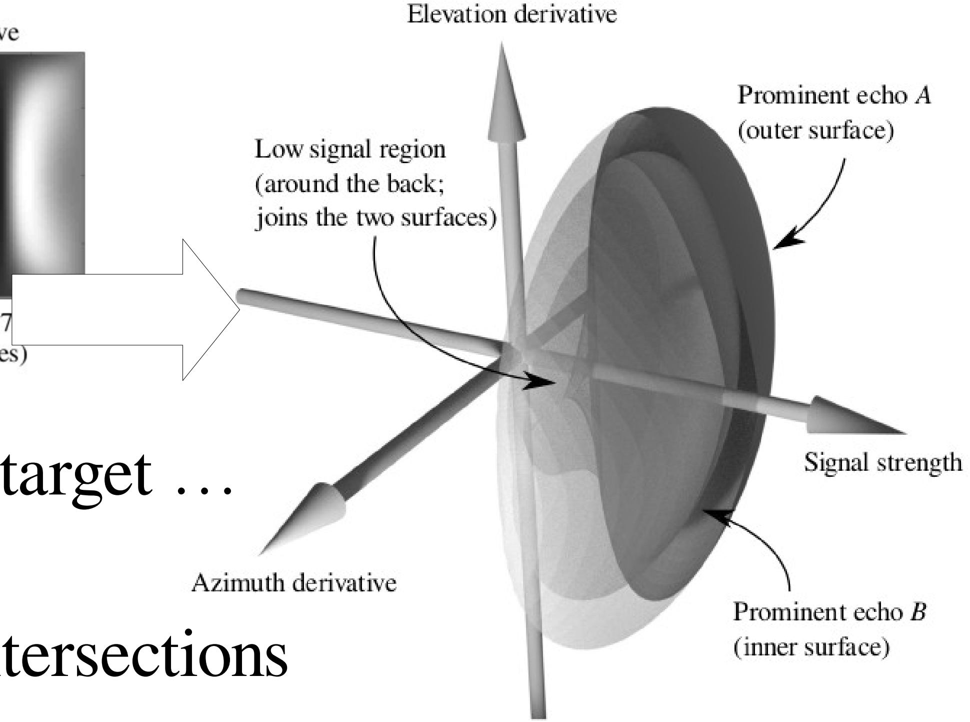
Lab check: Styrofoam cup with lid



Proposition 3. *Let M be a smooth manifold and assume that $n > (2 \dim M)$. For a generic v in $C^\infty_{\mathcal{D}}(M, \mathbb{R}^n)$, the self-intersections in the signature space of v only occur at points where $v(t) = 0$.*

An unexpected opportunity?





Az-el w/Doppler scan of a simulated target …

… also has no high SNR self intersections in signal space…

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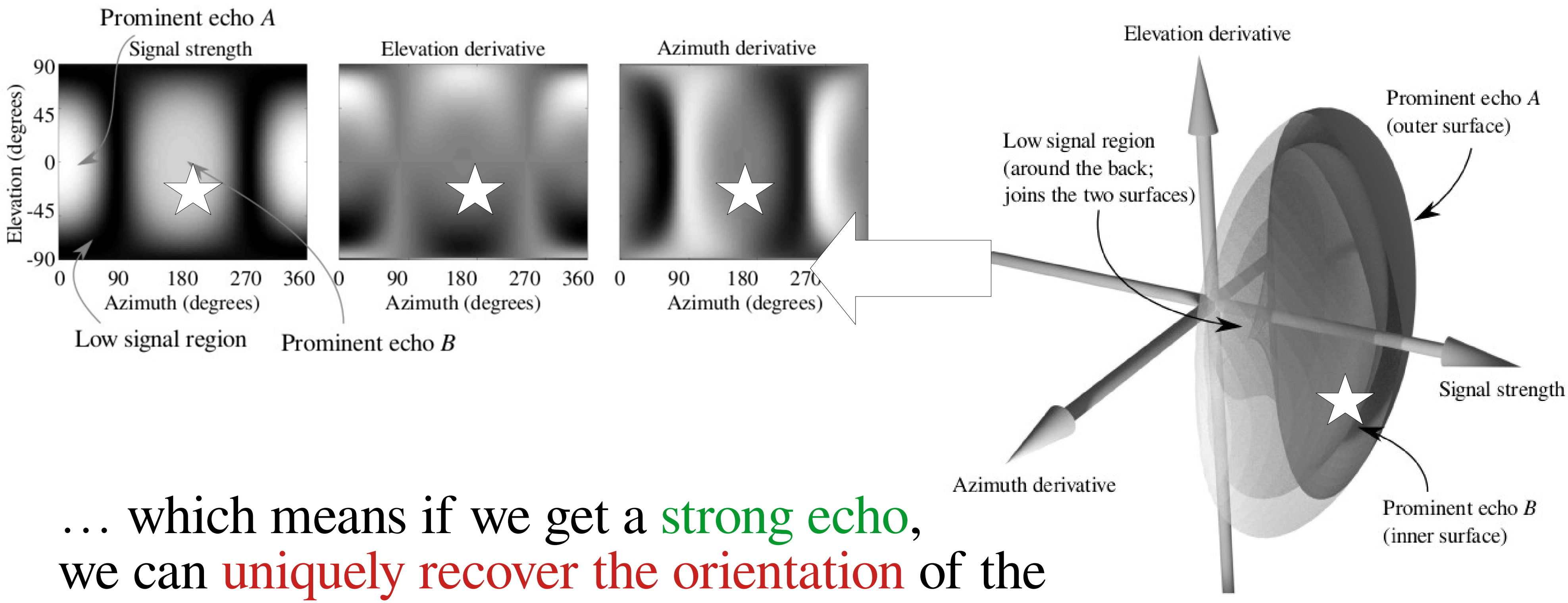

… which means if we get a strong echo, we can uniquely recover the orientation of the target **from one measurement!**

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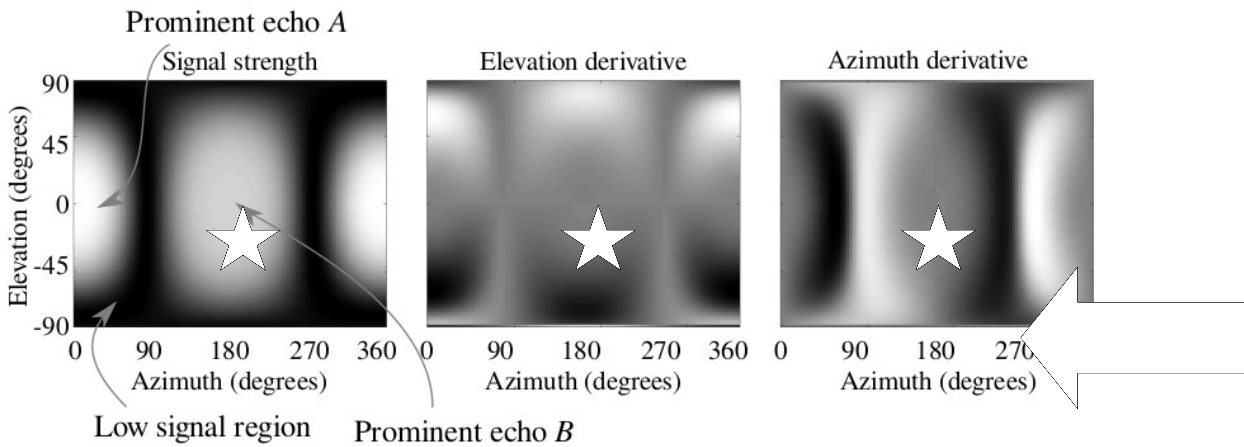
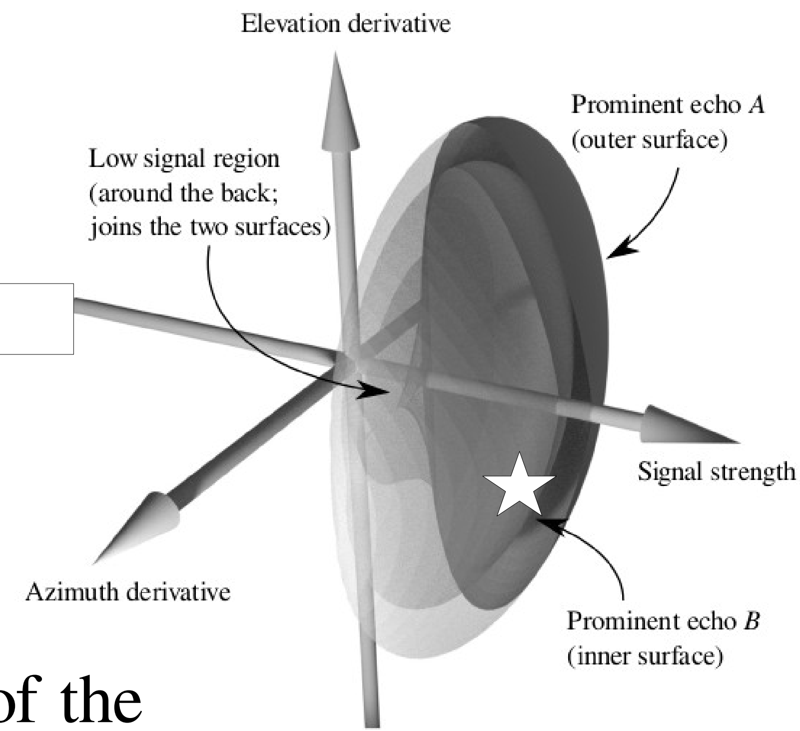




… which means if we get a strong echo, we can uniquely recover the orientation of the target **from one measurement!**

This is a **topological version** of *monopulse estimation*, which appears to be completely novel (esp. the Doppler part)

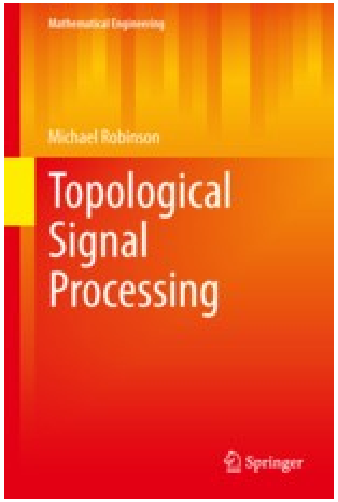
<u>Presently</u>: Implementing this idea on data from collaborators!

Michael Robinson

Next up

- Testing on real and realistic simulated data
  - Test bouquet-of-spheres hypothesis on collaborator-provided data
  - Test orientation-finding tools
- Tie back to topological structure theorems recently proven
  - Aim for geometric uncertainty quantification, especially around curvature, embedded reach, and the like...

To learn more...



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Relevant papers:

https://doi.org/10.1121/10.0037085

arXiv:2205.11311

https://doi.org/10.1017/S0956792522000365

Software:

https://github.com/kb1dds