## Finding cross-species orthologs with local topology

Michael Robinson


## Acknowledgements

- SRC: Chris Capraro
- PNNL:
- Cliff Joslyn
- Katy Nowak
- Brenda Praggastis
- Emilie Purvine


## Pacific Northwest

NATIONAL LABORATORY
Proudly Operated by Battelle Since 1965

- Baylor College of Medicine:
- Olivier Lichtarge
- Angela Wilkins
- Daniel Konecki

> Baylor
> College of Medicine

- Reza Ghanadan (DARPA/DSO SIMPLEX program)


## Problem statement

- Can we identify related proteins across species?


Species A
Species B
COG $=$ Clusters of Orthologous Groups - set of genetically related proteins
Michael Robinson

## Working dataset

- Source: StringDB version 9.1
http://string91.embl.de/
- Protein-protein interactions

Previous Knowledge

- Clusters of Orthologous Groups (COGs)
- 1133 species, 5214213 proteins, 143458 COGs
- Data extract: (Angela Wilkins and Daniel Konecki)
- 7 species: human, mouse, zebrafish, D. Melanogaster, C. Elegans, yeast, E. coli
- Only "experimentally confirmed" interactions
- 59010 proteins represented


## Protein-COG networks



## Using COG labels

- If two proteins are in the same COG, then they tend to be in other COGs together also

| ASIP | Specie | D2 |  |  | GB |  | COGS in PP | PPI-COG Network |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | human | 4 |  | 6.2 | 36 | 'COG0515' KOG0695', KOG360 | 'COG5023', 'COC | COG5040', KOG0290', 'K |
|  |  |  |  | 'KOG0841', 'KOG |  |  | KoGi37', 'KOG13888', KOG157 |
|  |  |  |  |  |  |  |  |
| 10090.ENS MUSP00000 105319 | mouse | 6 | 4. |  | 21 |  | 'COG0515', 'COG5023', 'COG5040,' KOG0290', 'KOG0657', KOG0695', 'KOG0841', 'KOG1375', 'KOG1388', 'KOG1574', 'KOG3606', 'KOG3656', 'KOG4222', 'KoG4643' |  |  |
|  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |

## Key insight

- If two proteins have
- similar interaction structure with neighboring proteins and
- their neighbors are in similar COGs

Then they probably are in the same COG

## Key insight

- If two proteins have
- similar interaction structure with neighboring proteins and
- their neighbors are in similar COGs

Then they probably are in the same COG

## Sheaf

Goal: "Zero in" on groups of proteins whose sequences are related, not to each other, but across species
Tool: Consistency radius of a sheaf of pseudometric spaces

## Base space

Goal: Narrow the search space of possible orthologs Tool: Local topological and geometric invariants

## What's new about this idea?

Usual procedure:

- Input:
- Sequence data
- Partial protein interactions
- No COG information
- Output:
- COG network

Our procedure:

- Input:
- Protein interactions
- Partial COG network
- No sequences
- Output:
- COG network


## Process flowchart



## Process flowchart



Michael Robinson

## Flag complex of PPI graph

- Vertices = proteins, Edges = interactions
- All cliques - an edge between every pair of vertices - become simplices


$$
v_{2}
$$

$$
\begin{array}{cc}
\text { Flagify } & \left\{v_{2}, v_{3}\right\} \\
\left\{v_{3}, v_{4}\right\} & \left\{v_{1}, v_{2}, v_{3}\right\} \\
\left.v_{1}, v_{3}\right\}
\end{array}
$$

Payoff: Better representation of multi-way interactions between proteins

## Matching metrics

- We look for pairs of proteins: one from each species with similar 2-hop neighborhoods
- There are several metrics available:

| Graph Metric | Description |
| :--- | :--- |
| Vertex degree histogram | A list of vertex degree frequencies |
| Adjacency spectrum | Eigenvalues of graph adjacency matrix |
| Graph Laplacian spectrum | Eigenvalues of the Laplacian matrix where a Laplacian matrix is the <br> adjacency matrix subtracted from the diagonal matrix of vertex degrees |
| Graph density (undirected <br> graph) | Density $=(2 m) /(n(n-1))$, where $n=$ \# edges, $m=$ \# vertices |
| Graph Betti number <br> (connected graph) | Graph Betti $=n-m+1$, where $n=$ \# edges, $m=$ \# vertices |

## Aside: Homology and spectra

- In a graph, the graph Laplacian $\Delta_{1}$ determines homology, so it's convenient and widely used



## Aside: Homology and spectra

- For cell complexes, the graph Laplacian and homology are different, but related
- There are "higher" Laplacians that determine homology, but they aren't much used* in data science


Geometry
Topology
Theorem (Hodge):
$\operatorname{ker} \Delta_{k} \cong H_{k}(C, \partial$.

* I'm not sure why, actually! But... we aren't either yet :-(

Michael Robinson

## Refining the search

- How well are local network invariants from a COG's proteins correlated across species?

| Graph Metric | Topological? | Pearson Correlation |
| :--- | :--- | :---: |
| Second bin degree histogram (D2) | Yes | 0.9046 |
| Second adjacency eigenvalue (A2) | Partially | 0.8823 |
| Second Laplacian eigenvalue (L2) | Partially | 0.3596 |
| Graph density (GD) | No | 0.5634 |
| Graph Betti number (GB) | Yes | 0.8840 |

Local topology is a strong indicator, but is not conclusive... Remember we're looking at 50000+ proteins!

- The local topology and geometry of the protein-COG network greatly reduces the search space


## Local sections

- The mantra of algebraic topology is "local to global"
- Poor scaling (usually cubic in the number of simplices)
- Requires linear algebra (usually good, but not always)
- Real data usually can't be globalized due to errors
- Very little effort has been expended by others about "partially global" results: local sections of sheaves
- We have recently been looking at local sections
- Discovery: Interesting combinatorics is present!
- Payoff: Partially global results are more realistic, and easier to compute


## Simplicial complexes

- An abstract simplicial complex consists of simplices (tuples of vertices)



## Simplicial complexes

- The attachment diagram shows how simplices fit together



## A sheaf is ...

- A set assigned to each simplex and ...



## Each such set is called the stalk over its simplex <br> $\mathbb{R}^{2}$ <br> This is a sheaf of vector spaces on a simplicial complex

## A sheaf is ...

- ... a function assigned to each simplex inclusion



## A sheaf is ...

- ... so the diagram commutes.



## Consider a vertex assignment

- Values are placed at vertices only, corresponding to protein metadata

$$
\begin{aligned}
& \binom{1}{0}
\end{aligned}
$$

## Consider a vertex assignment

- In some places there is consistency, but not all



## Maximal covers of local sections

- Theorem: (Praggastis) we can compute the cover algorithmically!

Question: What is the best cover by open sets, on each of which this assignment restricts to a section?

- Set of observations: $d(a, b)=1, d(b, c)=1.5, d(a, c)=2, d(c, e)=3$
- Max error (a radius): $\varepsilon^{*}=\max (d(a, b), d(b, c), d(a, c), d(c, e)) / 2=1.5$
- Sequence of radii: ( $0.5,0.75,1.0,1.5$ )
- Sectional filtration on $\varepsilon$

- Set of observations: $d(a, b)=1, d(b, c)=1.5, d(a, c)=2, d(c, e)=3$
- Max error (a radius): $\varepsilon^{*}=\max (d(a, b), d(b, c), d(a, c), d(c, e)) / 2=1.5$
- Sequence of radii: ( $0.5,0.75,1.0,1.5$ )
- Sectional filtration on $\varepsilon$
-0.0: a/b/c/e

- Set of observations: $d(a, b)=1, d(b, c)=1.5, d(a, c)=2, d(c, e)=3$
- Max error (a radius): $\varepsilon^{*}=\max (d(a, b), d(b, c), d(a, c), d(c, e)) / 2=1.5$
- Sequence of radii: ( $0.5,0.75,1.0,1.5$ )
- Sectional filtration on $\varepsilon$
$\square 0.0$ a/b/c/e
■ 0.5: ab/c/e

- Set of observations: $d(a, b)=1, d(b, c)=1.5, d(a, c)=2, d(c, e)=3$
- Max error (a radius): $\varepsilon^{*}=\max (d(a, b), d(b, c), d(a, c), d(c, e)) / 2=1.5$
- Sequence of radii: ( $0.5,0.75,1.0,1.5$ )
- Sectional filtration on $\varepsilon$
$\square 0.0$ a/b/c/e
■ 0.5: ab/c/e
- 0.75: ab/bc/e

- Set of observations: $d(a, b)=1, d(b, c)=1.5, d(a, c)=2, d(c, e)=3$
- Max error (a radius): $\varepsilon^{*}=\max (d(a, b), d(b, c), d(a, c), d(c, e)) / 2=1.5$
- Sequence of radii: (0.5,0.75, 1.0,1.5)
- Sectional filtration on $\varepsilon$
$\square 0.0 \mathrm{a} / \mathrm{b} / \mathrm{c} / \mathrm{e}$
- 0.5: ab/c/e
- 0.75: ab/bc/e
- 1.0: abc/e

- Set of observations: $d(a, b)=1, d(b, c)=1.5, d(a, c)=2, d(c, e)=3$
- Max error (a radius): $\varepsilon^{*}=\max (d(a, b), d(b, c), d(a, c), d(c, e)) / 2=1.5$
- Sequence of radii: (0.5,0.75, 1.0,1.5)
- Sectional filtration on $\varepsilon$
$\square 0.0 \mathrm{a} / \mathrm{b} / \mathrm{c} / \mathrm{e}$
- 0.5: ab/c/e
- 0.75: ab/bc/e
- 1.0: abc/e

c: HRadio BRadio


The consistency radius is the smallest threshold yielding global consistency Theorem: (Nowak) This can be computed algorithmically!

## Local PPI complexes

NB: we use the 2-hop neighborhood, even though I'm only showing the 1-hop neighborhood


## Joint local PPI complex

NB: we use the 2-hop neighborhood, even though I'm only showing the 1-hop neighborhood


## Sheaf of COG label sets

NB: we use the 2-hop neighborhood, even though I'm only showing the 1-hop neighborhood


## Sheaf of COG label sets

Three known COGs: $K, L, M$


## Sheaf of COG label sets

The COG database consists of a vertex assignment, like so... but this doesn't exhibit much self-consistency...


Species 1


Species 2
All restrictions are identity functions...

## Sheaf of COG label sets

... so instead assign the set of COGs of each protein and its neighbors...


All restrictions are identity functions...

## Sheaf of COG label sets

... Extend to maximal local sections. If not a global section...


All restrictions are identity functions...

## Sheaf of COG label sets

... compute the consistency radius
Use an appropriate set metric, for instance:


## Validation process



## Reciprocal BLAST validation



## RED- top hits

GREEN - within two
BLUE - within three


Less similar topology and
COG label structure

## Conclusions

- Consistency radius is a measure of relatedness of protein pairs
- 30-50\% of our "most likely" protein pairs are truly novel orthologs!
- Protein interaction network and COG self-consistency together predict sequence similarity
- Speculation: this is because important functional networks of proteins are preserved in evolution
- Maybe some of our protein pairs that don't have similar sequences are functionally similar?
- Maybe they play similar roles in different pathways?


## Next steps

- Further validation
- Finish processing all seven species we have data about
- Retrospective analyses... StringDB 9.1 is a year out of date
- Can we predict what was discovered over the past year?
- Sheaves seem natural to transfer information about model organisms, but are they actually effective?
- Extend processing to other metadata about the proteins in our network
- Drug interactions, diseases, and pathway networks (BioCyc repository, for instance)


## For more information

## Michael Robinson

## michaelr@american.edu

Preprints available from my website: http://www.drmichaelrobinson.net/


