# Finding cross-species orthologs with local topology





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### Problem statement

• Can we identify related proteins across species?





COG = Clusters of Orthologous Groups – set of genetically related proteins

# Working dataset

- Source: StringDB version 9.1 http://string91.embl.de/ Genomic
  - Protein-protein interactions



- Clusters of Orthologous Groups (COGs)
- 1133 species, 5214213 proteins, 143458 COGs
- Data extract: (Angela Wilkins and Daniel Konecki)
  - 7 species: human, mouse, zebrafish, D. Melanogaster, C. Elegans, yeast, E. coli
  - Only "experimentally confirmed" interactions
  - 59010 proteins represented



## Protein-COG networks



# Using COG labels

• If two proteins are in the same COG, then they tend to be in **other** COGs **together** also

Start Protein	Species	D2	A2	GB	COGS in PPI-COG Network
ASIP	human	4	6.2	36	'COG0515', 'COG5023', 'COG5040', 'KOG0290', 'KOG0657', 'KOG0695', 'KOG0841', 'KOG1375', 'KOG1388', 'KOG1574', 'KOG3606', 'KOG3656', ' <u>KOG4475</u> ', 'KOG4643'
10090.ENS MUSP00000 105319	mouse	6	4.2	21	'COG0515', 'COG5023', 'COG5040', 'KOG0290', 'KOG0657', 'KOG0695', 'KOG0841', 'KOG1375', 'KOG1388', 'KOG1574', 'KOG3606', 'KOG3656', ' <u>KOG4222</u> ', 'KOG4643'

• This holds for their neighbors as well

Only two differences



# Key insight

- If two proteins have
  - similar interaction structure with neighboring proteins and
  - their neighbors are in similar COGs
  - Then they probably are in the same COG



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### Base space

<u>Goal</u>: Narrow the search space of possible orthologs <u>Tool</u>: Local topological and geometric invariants

### Sheaf

<u>Goal</u>: "Zero in" on groups of proteins whose sequences are related, not to each other, but across species <u>Tool</u>: *Consistency radius* of a sheaf of pseudometric spaces



# What's new about this idea?

Usual procedure:

- Input:
  - Sequence data
  - Partial protein interactions
  - No COG information
- Output:
  - COG network

Our procedure:

- Input:
  - Protein interactions
  - Partial COG network
  - No sequences
- Output:
  - COG network



### Process flowchart



### Process flowchart



# Flag complex of PPI graph

- Vertices = proteins, Edges = interactions
- All *cliques* an edge between every pair of vertices become simplices



Payoff: Better representation of multi-way interactions between proteins



# Matching metrics

- We look for pairs of proteins: one from each species with similar 2-hop neighborhoods
- There are several metrics available:

Graph Metric	Description
Vertex degree histogram	A list of vertex degree frequencies
Adjacency spectrum	Eigenvalues of graph adjacency matrix
Graph Laplacian spectrum	Eigenvalues of the Laplacian matrix where a Laplacian matrix is the adjacency matrix subtracted from the diagonal matrix of vertex degrees
Graph density (undirected graph)	Density = $(2m) / (n(n-1))$ , where $n = \#$ edges, $m = \#$ vertices
Graph Betti number (connected graph)	Graph Betti = $n - m + 1$ , where $n = #$ edges, $m = #$ vertices



# Aside: Homology and spectra

• In a graph, the graph Laplacian  $\Delta_1$  determines homology, so it's convenient and widely used





# Aside: Homology and spectra

- For cell complexes, the graph Laplacian and homology are different, but related
- There are "higher" Laplacians that determine homology, but they aren't much used\* in data science



# Refining the search

• How well are local network invariants from a COG's proteins correlated across species?

Graph Metric	Topological?	Pearson Correlation
Second bin degree histogram (D2)	Yes	0.9046
Second adjacency eigenvalue (A2)	Partially	0.8823
Second Laplacian eigenvalue (L2)	Partially	0.3596
Graph density (GD)	No	0.5634
Graph Betti number (GB)	Yes	0.8840

Local topology is a strong indicator, but is not conclusive... Remember we're looking at 50000+ proteins!

• The local topology and geometry of the protein-COG network greatly reduces the search space



# Local sections

- The mantra of algebraic topology is "local to global"
  - Poor scaling (usually cubic in the number of simplices)
  - Requires linear algebra (usually good, but not always)
  - Real data usually can't be globalized due to errors
- Very little effort has been expended by others about "partially global" results: *local sections* of sheaves
- We have recently been looking at local sections
  - Discovery: Interesting combinatorics is present!
  - Payoff: Partially global results are more realistic, and easier to compute



# Simplicial complexes

• An *abstract simplicial complex* consists of *simplices* (tuples of vertices)





# Simplicial complexes

• The *attachment diagram* shows how simplices fit together





A sheaf is ...

• A set assigned to each simplex and ...



Each such set is called the *stalk* over its simplex

 $\mathbb{R}^{3}$ 

 $\mathbb{R}$ 

This is a sheaf **of** vector spaces **on** a simplicial complex

A sheaf is ...

• ... a function assigned to each simplex inclusion





A sheaf is ...

• ... so the diagram commutes.





# Consider a vertex assignment

• Values are placed at vertices only, corresponding to  $\begin{pmatrix} 1\\ 0 \end{pmatrix}$ protein metadata ?  $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix} (1 & 0) \\ \begin{pmatrix} 2 \\ 0 \\ 0 \\ 0 \\ 1 & 0 \\ 2 & 1 & 0 \end{pmatrix} (1 & 0) \\ \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 1 & 0 \end{pmatrix}$  $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ \end{pmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ \end{bmatrix} \begin{pmatrix} 1 & 0 & 2 \\ 2 & 1 & -1 \\ \end{pmatrix}$ 



## Consider a vertex assignment

• In some places there is consistency, but not all





# Maximal covers of local sections

• <u>Theorem</u>: (Praggastis) we can compute the cover algorithmically!  $\begin{pmatrix} 1\\ 0 \end{pmatrix}$ <u>Question</u>: What is the best cover by open sets, on each  $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$  $\begin{pmatrix} 2\\ 0 \end{pmatrix}$ of which this assignment restricts to a section?  $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}$ (10) $\binom{100}{2}$ 





- Set of observations: d(a,b)=1, d(b,c)=1.5, d(a,c)=2, d(c,e)=3
- Max error (a radius):  $\varepsilon^* = \max(d(a,b), d(b,c), d(a,c), d(c,e))/2 = 1.5$
- Sequence of radii: (0.5,0.75,1.0,1.5)
- Sectional filtration on  $\varepsilon$





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0.0: a/b/c/e

0.5: ab/c/e





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- Sectional filtration on  $\varepsilon$ 
  - 0.0: a/b/c/e
  - 0.5: ab/c/e
  - 0.75: ab/bc/e









The *consistency radius* is the smallest threshold yielding global consistency <u>Theorem</u>: (Nowak) This can be computed algorithmically!

# Local PPI complexes

NB: we use the 2-hop neighborhood, even though I'm only showing the 1-hop neighborhood



# Joint local PPI complex

NB: we use the 2-hop neighborhood, even though I'm only showing the 1-hop neighborhood





NB: we use the 2-hop neighborhood, even though I'm only showing the 1-hop neighborhood









... so instead assign the set of COGs of each protein and its neighbors...



... Extend to maximal local sections. If not a global section...





### Validation process



# Reciprocal BLAST validation



# Conclusions

- Consistency radius is a measure of relatedness of protein pairs
  - 30-50% of our "most likely" protein pairs are truly novel orthologs!
  - Protein interaction network and COG self-consistency together predict sequence similarity
- Speculation: this is because important functional networks of proteins are preserved in evolution
  - Maybe some of our protein pairs that don't have similar sequences are functionally similar?
  - Maybe they play similar roles in different pathways?



# Next steps

- Further validation
  - Finish processing all seven species we have data about
  - Retrospective analyses... StringDB 9.1 is a year out of date
  - Can we predict what was discovered over the past year?
- Sheaves **seem** natural to transfer information about model organisms, but are they actually effective?
  - Extend processing to other metadata about the proteins in our network
  - Drug interactions, diseases, and pathway networks (BioCyc repository, for instance)



### For more information

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